

8412

NASA TT-F - Number

7-8412

FACILITY FORM 802

N67 10182	
(ACCESSION NUMBER)	
26	(THRU)
(PAGES)	
	(CODE)
(NASA CR OR TMX OR AD NUMBER)	13
	(CATEGORY)

THERMAL RADIATION EFFECTS IN THE UPPER ATMOSPHERE

K. Y. Kondrat'yev

Paper presented at the COSPAR Symposium on the Physics of the Upper Atmosphere, Moscow, 1967

GPO PRICE \$ _____

CFSTI PRICE(S) \$ _____

Hard copy (HC) 2.00

Microfiche (MF) 1.50

K. Y. Kondrat'yev

"Thermal Radiation Effects in the Upper Atmosphere"

Soon after the discovery of the stratosphere at the beginning of this century, a number of theories were proposed explaining the isothermal conditions in the stratosphere by the existence of a state of radiative equilibrium in this layer.

Since then, the atmosphere has been traditionally divided into the "lower" (convection zone), and the "upper" (radiation zone).

Our knowledge about the principles of the thermal regime of the upper atmosphere has greatly increased in the last decades and, especially, in recent years.

The fact establishing that the atmosphere is in a state of turbulent mixing up to an altitude of about 100 km is of great importance in this respect. From this it follows that radiation is not the only factor controlling the thermal regime of the stratosphere and the thermosphere. The situation in the thermosphere is similar as regards thermal conductivity.

The variety of mechanisms determining the thermal regime of the upper part of the thermosphere and the exosphere is extremely great.

Despite the facts mentioned above, the conclusion that radiation is one of the major factors in the thermal regime of the upper atmosphere still holds.

It is for this reason that we are discussing today the problem of thermal radiation effects in the upper atmosphere.

The problem under consideration is very broad and diversified. We shall attempt to review this problem without going into a detailed analysis of it. On the contrary, our purpose is to present certain views on the thermal effects of radiation in the upper atmosphere based only on investigations conducted in this field by the Department of Atmospheric Physics at Leningrad University.

In order to solve the problem of the thermal regime in the upper atmosphere, we must first of all keep in mind the essential characteristics of the processes taking place in different atmospheric layers.

The division of the upper atmosphere into the stratosphere, thermosphere, and exosphere means, in particular, that there are several zones which have a specific thermal regime.

Numerous computations and, in part, direct measurements [see (1,2)] show that extensive regions of the stratosphere and mesosphere are in a state close to radiative equilibrium.

To illustrate this, we shall present some of the results of complex investigations on the radiation balance in the troposphere and stratosphere conducted at Leningrad University.

We have developed a complex of automatic balloon instruments which make it possible to measure the radiation balance, its components, the main meteorological elements (temperature, pressure, and air humidity) and the ozone content in the atmosphere during the daytime.

Fig. 1. gives the results of measurements made on November 14, 1961, which clearly show that the radiative equilibrium zone is found above the 9-km level. Even though we know that the influence of turbulent mixing may be rather important at this level, it is not possible yet reliably and fully to account for this influence, since our knowledge of the quantitative characteristics of turbulence in the stratosphere and mesosphere is insufficient. Hence we may in turn consider the problem of the vertical temperature distribution in the stratosphere and mesosphere within the framework of the theory of the radiative equilibrium by utilizing contemporary notions concerning radiative transfer in the upper atmosphere. Unfortunately, the little information available on humidity in the stratosphere prevents us from making reliable quantitative calculations of the temperature distribution in this layer.

That is why G.M. Shved in a work which we shall discuss later takes into consideration the temperature distribution in the mesosphere, i.e. at altitudes from 30 km to 80 km.

It is assumed that during the day absorption of solar radiation by molecular oxygen and ozone ordinarily is equivalent to long-wave radiation in the 15μ CO_2 band and the 9.6μ ozone band in the mesosphere.

Our purpose is to calculate the constant vertical temperature distribution based on these assumptions.

The mesosphere is divided into n layers. For each layer the condition of radiative equilibrium is put down. The intensity of black body radiation $B_\nu(T)$ is approximated by a Taylor's power series

$$B_\nu(T) = \sum_{l=0}^m b_{\nu l} \theta^l, \quad (1)$$

where $b_{\nu l}$ are the coefficients of the series. The spectral region under

consideration is divided into a number of intervals (index r) in which $B_\nu(T)$ may be assumed to vary only slightly.

The temperature distribution is found by solving n equations with respect to n unknowns of the θ_i type:

$$h_i = \sum_{k=1}^n \sum_{l=0}^n \bigwedge_{ik}^l \theta_k^l, \quad i=1, 2, \dots, n, \quad (2)$$

where

$$h_i = h_s(z_i) + 2 \sum_r [J_{r0} \int_r d\nu k_\nu(z_i) E_2(z_i, 0; \nu); \quad (3)$$

$$\Lambda_{ik}^1 = \begin{cases} -2(\Delta z)_k \sum_r \tilde{b}_{r1} \int_r d\nu k_\nu(z_i) k_\nu(z_k) E_1(z_i, z_k; \nu), & i < k \\ 4 \sum_r \tilde{b}_{r1} \int_r d\nu k_\nu(z_i), & i = k, \\ -2(\Delta z)_k \sum_r \tilde{b}_{r1} \int_r d\nu k_\nu(z_i) k_\nu(z_k) E_1(z_k, z_i; \nu), & i > k \end{cases} \quad (4)$$

Here $k_\nu(z)$ is the absorption coefficient of long-wave radiation at frequency ν_∞ and at altitude z ;

$$E_n(a, b; \nu) = \int_1^\infty dt \cdot t^{-n} e^{-t} \int_a^b k_\nu(\zeta) d\zeta$$

$(\Delta Z)_k$ is the thickness of the layer in which temperature is considered to be constant and is equal to T_k ;

$h_s(z)$ is solar heating for a unit volume (it depends on the angular solar altitude and the O_2 and O_3 content);

J_{r0} is infrared radiation in the space r between the surface of the earth and the atmosphere below the level accepted as zero;

$$\tilde{b}_{r1} = \begin{cases} \frac{b_{r1}}{\lambda(z)} - \frac{h_s(z)}{4\pi \int d\nu k_\nu(z)}, & l \geq 1 \\ b_{r1}, & l = 0 \end{cases} \quad (5)$$

where b_{r1} is determined from (I). The second item in b_{r1} is related to the deviation from emission according to Kirchhoff's law;

θ is the lifetime of the excited vibrational state of a molecule;
 λ is the time required for the vibrational states to return to normal by collision. Since $\lambda \sim \frac{1}{p}$ (p is atmospheric pressure), a cor-

rection for the deviation from Kirchhoff's law is significant for the 15μ band of CO_2 above an altitude of 70 km. This correction is not

essential for O_3 , since the contribution of O_3 to cooling is negligibly small from 60 km up in comparison to the contribution of CO_2 .

Thus, the problem boils down to calculating the following integrals for all r intervals:

$$\left. \begin{aligned} \int_r d\nu k_\nu(z_i) E_2(z_i, 0; \nu) , \\ \int_r d\nu k_\nu(z_i) k_\nu(z_k) E_1(z_i, z_k; \nu) \end{aligned} \right\} \quad (6)$$

Above 25-30 km for the P and R components of the vibrational-rotational 15μ band of CO_2 and 9.6μ band of O_3 we may assume that we have an isolated line.

The Lorentz half-width $\Delta_L^\nu \sim p$ and the lines become so narrow

that their overlapping can be neglected.

For the Q component, the overlapping of the lines is very essential, with the possible exception of the upper mesosphere. Therefore the question concerning the Q component requires special study.

However, even in this case we may expect satisfactory results by approximation on the basis of an isolated line.

When the lines overlap significantly, the region of the Q component becomes practically opaque and "does not take part" in radiational heat exchange and, therefore, the contribution of the Q component to radiative heating is small. When overlapping is not very great, approximation on the basis of an isolated line can be used.

The integrals in (6) must be the sums of the following integrals according to frequencies belonging to the individual lines:

$$\left. \begin{aligned} \int_{-\infty}^{\infty} d\nu k_\nu(z_i) E_2(z_i, 0; \nu) , \\ \int_{-\infty}^{\infty} d\nu k_\nu(z_i) k_\nu(z_k) E_1(z_i, z_k; \nu) . \end{aligned} \right\} \quad (7)$$

The number of these integrals is determined by the number of lines in the r interval. Integrals (7) must be calculated for a set of S lines of different intensities.

For each actual line with a certain intensity S , integrals (7) are considered equal to the value computed for that intensity of the line which is closest to the given actual intensity.

The calculations are made by taking into account the complex Doppler-Lorentz shape of the line.

Therefore, it is important to take into account the dependence of

(7) on temperature. The Lorentz half-width is $\Delta_L^\nu \sim \frac{1}{\sqrt{T}}$ and the Doppler half-width is $\Delta_D^\nu \sim \sqrt{T}$. This dependence is evidently not essential for

the range of atmospheric temperatures. However, the temperature dependence of the intensity of the line must also be taken into account

$$S(T) \sim \frac{1}{T} e^{-\frac{E_v}{kT}}, \quad (8)$$

where E_v is the vibrational energy of the molecule corresponding to the lower state of vibrational transition; k is Boltzmann's constant.

Since there is reason to expect that the calculated temperature distribution will qualitatively correspond to the observed distribution, integrals (7) can be computed by using the standard vertical temperature distribution (all other structural parameters of the atmosphere are given in the same way). Note that calculation of integrals (7) by using the real temperature distribution has an independent value; for example, for computing the radiative heating of the atmosphere. Integrals (7) are also calculated for a simple case where S is independent of T .

The last calculation is applicable only to the main vibrational transition of the 15μ band of CO_2 which contributes not less than 50%

of the total contribution of this band to the cooling of the mesosphere. If the volume concentration of CO_2 is considered constant, then that of O_3 changes greatly with altitude and must be taken into account in

integrals (7) for the O_3 lines. By calculating the integrals by frequency intervals of finite width (the limits of integration are $\nu_0 - \nu_1$ and $\nu_0 + \nu_1$, where ν_0 is the frequency of the center of the line and ν_1 is the distance from the center) and comparing them with the full integral, it is possible to determine the applicability of the approximation based on an isolated line from the known mean distance between the lines of the band.

It is difficult to calculate integral (7) because there is a large number of lines and because many different calculations are involved. Therefore, an electronic computer must be used. After (7) is calculated, we shall need a high-speed computer to solve the rather complicated system of equations (2).

It is useful to calculate temperature for a number of J_{r0} and $h_s(z)$ values to establish the dependence of temperature distribution on seasons and latitude, on temperature distribution in the lower atmosphere, on cloudiness, etc. We must note that the considered formalism of the temperature calculation in the framework of the radiative

equilibrium theory differs somewhat from the (not very accurate) method of calculating temperature which takes into account turbulent and molecular heat conductivity, if the turbulent and molecular heat exchange coefficients are known.

An almost trivial generalization of the results mentioned above is achieved when the influence of thermal conductivity is taken into consideration.

At the same time we shall calculate the following integrals which are pertinent to the theory of radiation transfer:

$$\int_{-\infty}^{\infty} d\nu \left[1 - e^{-\int_a^b k_\nu(3) d3} \right],$$

$$\int_{-\infty}^{\infty} d\nu [1 - 2E_3(a, b; \nu)],$$

$$\int_{-\infty}^{\infty} d\nu k_\nu(a) E_2(a, b; \nu),$$

$$\int_{-\infty}^{\infty} d\nu k_\nu(b) E_2(a, b; \nu).$$

The programming of calculations on the M-20 electronic computer has now been completed, and some of the calculations have been made.

The average time it takes to calculate one integral is from half a minute to two minutes.

As the solution of the problem requires the calculation of a large number of integrals it was expedient to determine the possibility of using the approximation of the intense line (see [1]).

By comparing the integrals computed accurately with those computed by the approximate formulas, we established the limits of applicability of the given approximation for the 15μ band of CO_2 .

If $S > 0.3 \text{ cm}^{-1} (\text{cm-atm})^{-1}$, which includes approximately half of the considered lines of the main vibrational transition, then the integral

$$\int_0^{\infty} d\nu \left[1 - e^{-\int_a^b k_\nu(3) d3} \right],$$

which represents radiation absorption, may be calculated within 1 percent of accuracy or better by the approximate formula for atmospheric layers with a lower boundary $a > 45 \text{ km}$.

The limits of applicability of the approximation evidently depend upon the thickness of the $b-a$ layer.

For the most probable a the given accuracy requires a layer thickness of not less than 3 km.

With decreasing S , the range of application of the approximate formula decreases rapidly.

At $S \approx 0.03 \text{ cm}^{-1} (\text{cm-atm})^{-1}$, sufficient accuracy can be obtained only for layers with $a \leq 35 \text{ km}$.

The same is true when the approximation of the intense line is used for calculating the integrals

$$\int_0^{\infty} d\nu k_{\nu}(a) k_{\nu}(b) E_2(a, b; \nu),$$

representing the contribution of the atmosphere at the "a" level to heating at the "b" level (or vice versa).

However, in this case the range of application of the approximate formulas is somewhat narrower and depends, in particular, on the position of the upper boundary of the b layer.

For intensities of 3, 0.3, and $0.03 \text{ cm}^{-1} (\text{cm-atm})^{-1}$, the approximate maximum values of a are 45.35 and 30 km, respectively, and the corresponding maximum possible values of b are 90.85 and 65 km.

It is more likely that a less intense line contributes less to heating than a more intense one.

But it was found that for certain optical thicknesses the decrease of S in a given range causes not only a reduction in the decrease, of the integral values, but also an increase after which the values of the integrals decrease again.

Here are some examples:

- 1) heating of the atmosphere at 65 km from the layer near 25 km at

$S = 0.0003 \text{ cm}^{-1} (\text{cm-atm})^{-1}$ is nearly the same as at $S = 0.3 \text{ cm}^{-1} (\text{cm-atm})^{-1}$;

- 2) heating at 80 km from the atmospheric layer near 40 km at

$S = 0.003 \text{ cm}^{-1} (\text{cm-atm})^{-1}$ is 10 times greater than that at $S = 3 \text{ cm}^{-1} (\text{cm-atm})^{-1}$.

It considerably increases the role of the lines of low intensity in the thermal regime of the mesosphere, especially for the layer from 60 to 90 km.

The reason for this phenomenon evidently is the change in the shape of the line along the path of a ray.

Emission from below due to the near Lorentz shape and the intense wings of the lines occurs in spectral intervals which are at some distance from the center of the line.

Absorption at the upper levels depends on lines with a shape close to Doppler's whose width hardly changes in the atmosphere and whose wings are weak.

The decrease in intensity results in a shift of the spectral intervals where absorption occurs towards the center of the line.

Therefore, the emission coming from below is absorbed not only by the weak wings but also by the intense central part of the line.

So "the effect of the shift" does not only compensate for, but also exceeds the effect of a decrease in the intensity of the line.

From this it follows that it is of primary importance to take into account the change in the form of the spectral line with elevation in the problem of radiative flux divergence in the mesosphere.

Now we shall discuss the temperature distribution in the thermosphere which is being studied by Y.P. Suslov. Calculations of the vertical temperature distribution are based on the following assumptions:

1. The source of heating is solar radiation which is absorbed in the thermosphere at wavelengths $\lambda < 1700 \text{ \AA}$ (especially the emission lines at wavelengths $\lambda = 300-800 \text{ \AA}$, in particular, the helium lines 304 \AA and 584 \AA).
2. The temperature distribution depends on the relationship between the divergence of the thermal energy flux as a result of the absorption of solar radiation and its loss by molecular heat conductivity and the natural infrared emission of the thermosphere.
3. The thermosphere is in a state of thermal equilibrium (the temperature of electrons is approximately equal to that of atoms and molecules, the natural infrared emission is thermal).

If we accept these assumptions, the temperature distribution in the thermosphere can be represented by the thermal conductivity equation

$$\frac{d}{dz} \left(\lambda \frac{dT}{dz} \right) + q(z) - l(z) = 0. \quad (9)$$

where T is temperature, q is the amount of thermal energy received by 1 cm^3 per sec at a level z as the result of the absorption of ultraviolet solar radiation; l - is the natural emission of 1 cm^3 per sec at the z level; $\lambda = a T^{3/4}$ is the thermal conductivity coefficient (a is a constant, somewhat dependent on the chemical composition of the atmosphere).

Ultraviolet radiation is absorbed in the continuous spectrum (dissociation and photionization); therefore, the weakening of solar radiation in the thermosphere is exponential. The intensity of incident radiation at the z level is equal to

$$J(z) = J_0 e^{-\sigma \sec \theta \int_z^{\infty} n dz}, \quad (10)$$

where J_0 is the solar radiation flux on 1 cm^2 per sec at the boundary of the atmosphere;

σ is the absorption cross-section, θ is the angle of the sun at zenith, $\int_z^{\infty} n dz$ is the amount of radiation absorbing particles in an atmospheric column from the z level up to ∞ .

It has been assumed that the distribution of particles with height is expressed by the barometric formula

$$n(z) = \frac{n_0 T_0}{T} \exp \left(- \int_{z_0}^z \frac{gm}{kT} dz \right). \quad (11)$$

Substituting (11) in (10) we get

$$J(z) = J_0 e^{-\sigma \sec \theta \int_z^{\infty} \frac{n_0 T_0}{T} \exp \left(- \int_{z_0}^{z'} \frac{gm}{kT} dz' \right) dz}. \quad (12)$$

The quantity of energy absorbed by 1 cm^2 in 1 sec is equal to

$$\frac{dJ(z)}{dz} = -\sigma \sec \theta n(z) J(z). \quad (13)$$

So, without considering the losses due to emission, formula (9) can be represented in the form:

$$\frac{d}{dz} \left(\lambda \frac{dT}{dz} \right) + \sigma n(z) \sec \theta \cdot J(z) = \frac{d}{dz} \left(\lambda \frac{dT}{dz} + J(z) \right) = 0 \quad (14)$$

or

$$\lambda \frac{dT}{dz} + J(z) = C.$$

The constant C is determined by assuming that $\frac{dT}{dz} = 0$ at the boundary of the atmosphere (i.e. that there is no heat flux here due to thermal conductivity), and that $J(\infty) = J_0$. Then $C = J_0$, and equation (14) is written in the form

$$\lambda \frac{dT}{dz} - J_0 (1 - e^{-\sec \theta \int_z^\infty n(z) dz}) = 0, \quad (15)$$

where $n(z)$ is given by (11).

$$\text{The value } Q = J_0 [1 - e^{-\sigma \sec \theta \int_z^\infty n(z) dz}]$$

is the energy absorbed in an atmospheric column from the z level to ∞ (taking into account that different lines of the incident radiation have different intensities and different absorption cross-sections, this value, in reality is equal to

$$Q = \sum_i J_{0i} [1 - e^{-\sigma_i k \sec \theta \int_z^\infty n_k(z) dz}],$$

where summation is performed according to the wavelengths as well as to atmospheric components). Taking into consideration the losses of energy due to thermosphere emission, we can present formula (9) in the following form:

$$\lambda \frac{dT}{dz} + \sum_{i,k} J_{0i} (1 - e^{-ik \sec \theta \int_z^\infty n_k dz}) - L = 0 \quad (16)$$

where $L = \int_z^\infty l(z) dz.$

As there are no molecules with a dipole moment in the thermosphere, its own radiation is small. As stated by D.R. Bates, transition

$O(P_1^3) \rightarrow O(P_2^3) + h\nu (0.020 \text{ eV})$ of atomic oxygen plays the chief role

in the thermal emission of the thermosphere. Therefore, if we assume that the medium is optically thin, the thermal emission of the atmospheric column from altitude z to ∞ is equal to

$$L(z) = AP \int_z^\infty n_o(z) dz, \quad (17)$$

where A - is the probability of transition, P is a function giving the number of atoms in the excited state for a given transition*, and $n_0(z)$ is the amount of oxygen atoms in 1 cm^3 .

The values λ , L , and Q depend on temperature T which is an unknown function of Z . The dependence of density on temperature is very important. Therefore, in order to find T from equation (16), we must use the method of successive approximations, in which Q , L and λ are computed first for the given temperature distribution, and then T is determined from (16). The temperature distribution obtained is used again to determine Q , L , and λ , then the temperature is computed once more, etc.

The solar spectrum in the extreme ultraviolet region is not sufficiently known yet; in particular the important helium lines 304 \AA and 584 \AA . Nor are the values of the absorption cross-sections well known.

The composition of the thermosphere at different altitudes is not known for certain either.

The lack of these data does not make it possible to find the exact solution of the stated problem.

However, it is possible to find some common features of the temperature distribution at different altitudes in the thermosphere based on the assumptions made earlier, and the dependence of this distribution on the main parameters (incident energy, the absorption coefficients, and density).

The calculations were made by representing incident radiation by two lines having absorption cross-sections of about $1 \cdot 10^{-18} \text{ cm}^2$, corresponding to maximum absorption at an altitude of 100-120 km (E layer, continuous Schumann-Runge X-ray radiation), and $1 \cdot 10^{-17} \text{ cm}^2$ with maximum absorption at an altitude of 160-180 km (F layer, helium lines 308 \AA and 584 \AA , and other lines at $\lambda < 800 \text{ \AA}$).

The results of our calculations lead to the following conclusions:

1. The temperature gradient in the thermosphere is positive everywhere and decreases evenly with altitude. This results in the appearance of a practically isothermal region in the upper part of the thermosphere at an altitude of 260-300 km.

* Assuming equilibrium distribution, we get

$$P = \frac{g_1 \exp(-e_1/kT)}{g_2 + g_1 \exp(-e_1/kT) + g_0 \exp(-e_0/kT)},$$

where g_2 , g_1 , g_0 are statistical weights, and ϵ_1 , ϵ_0 are the energies of the corresponding states.

The losses of thermal energy in the entire thermosphere as a result of emission are less than the energy absorbed, i.e. $Q - L > 0$ in the entire thermosphere (fig.2).

2. In order for the temperature to rise to about 1200-1800°K in the isothermal region, the energy absorbed and converted into heat must reach 1.5-2.5 erg/cm² sec.

This means that in order to maintain the foregoing temperature in the thermosphere, minimum solar radiation energy must be 3.5-5.6 erg/

/cm² sec, and 1.2-2 erg/cm² sec must be provided by radiation of $\lambda < 800 \text{ \AA}$ and absorbed in the F-layer.

3. Most of the energy is absorbed below 200 km. The greatest temperature gradients are found below 150-170 km, ranging from 17° K/km with an

energy of 1.5 erg/cm² sec to 30° K/km with an incident energy of 2.5

erg/cm² sec at an altitude of 120 km.

An increase in density, or in absorption cross-sections, results in an increase of the maximum absorption level and, consequently, in higher temperatures at the same incident energy.

4. The greater the radiation absorbed in the F layer, the higher the temperature in the middle and upper thermosphere.

5. The amount of energy absorbed per cm² diminishes at oblique incidence and the length of the path along which absorption takes place increases, i.e. the maximum absorption altitude increases.

These two factors oppose each other, but the influence of the first is greater and, as a result, the temperature of the thermosphere decreases from the lower towards the middle latitudes.

The problem of the temperature distribution in the thermosphere was solved by O.P. Filipovich in a different manner.

Estimates of the role of different factors in the heat regime of the upper atmosphere are given in monograph [2].

This monograph also discusses a method of theoretically determining the temperature distribution in the upper atmospheric layers.

Here, the equation of energy conservation or the generalized thermal conductivity equation was used as a starting principle.

Further the thermal conductivity equation was used to actually calculate the vertical temperature distribution in the thermosphere (i.e. at altitudes above 100 km).

However, in addition to the method of determining $T(z)$, from the equation of energy conservation, there is another method that may be used to solve this problem.

The temperature distribution can be determined from empirical data on the density of the atmosphere (which have been obtained chiefly by means of rockets and satellites [3]) by using the equation of hydrostatic equilibrium ($dp = -gdz$) combined with the equation of state of an ideal gas ($p = nkT$).

The second method is often used in constructing models of the upper atmosphere.

1. Determining the Temperature of the Thermosphere from the Thermal Conductivity Equation

The equation for average temperature

$$\frac{d}{dz} \left(\lambda \frac{d\bar{T}}{dz} \right) + \bar{q}(z) - \bar{l}(z) = 0 \quad (18)$$

was taken as a starting point, where λ is the thermal conductivity coefficient, $(\bar{q} - \bar{l})$ is the radiative energy converted into thermal energy, written as the difference between absorbed energy (\bar{q}) and radiation (\bar{l}) of 1 cm^3 of gas per second.

Equation (18) is derived from the more accurate equation of thermal conductivity

$$\frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) + (q - l) = \frac{\partial(cT)}{\partial t}, \quad (19)$$

where c is the heat capacity of a unit volume if the temperature is expressed as

$$T(z, t) = \bar{T}(z) + \Delta T(z, t), \quad (20)$$

where $\Delta T(z, t)$ are the temperature fluctuations [2, ch. IX].

Since thermal conductivity $\lambda = \lambda(z, T)$ depends on temperature, $T(z)$ can be determined from equation (18), for instance, by the method of successive approximation which was used by F.S. Johnson (see [2]).

However, the solution of equation (18) can be considerably simplified if we represent the thermal conductivity coefficient by the following expression:

$$\lambda(z) = A(z) T^S(z), \quad (21)$$

where function $A(z)$ depends on the composition of the atmosphere.

Taking into account (21), the complete solution of equation (18) can be written as

$$\bar{T}(z) \left\{ (s+1) \left[\bar{T}^s(z_0) \frac{\partial \bar{T}(z_0)}{\partial z} A(z_0) \int_{z_0}^z \frac{dz'}{A(z')} = \int_{z_0}^z \frac{dz'}{A(z')} \int_{z_0}^{z'} (\bar{q} - \bar{l}) dz'' \right] \right\}^{\frac{1}{s+1}}, \quad (22)$$

where z_0 and z'_0 are the lower and the upper boundaries of the region under consideration.

Solution (22) is the initial formula from which the vertical temperature distribution has been calculated.

As we can see from (22), the following data are needed to determine $T(z)$:

1. The distribution of the radiative heat flux divergence with altitude $(\bar{q}-1)$.
2. The dependence of $A(z)$ on altitude.
3. The average temperature $T(z_0)$ at the initial boundary of the layer under consideration.
4. The mean heat flux at the upper boundary of the layer

$$Q(z'_0) = \bar{T}^s A(z'_0) \frac{\partial \bar{T}(z'_0)}{\partial z} = \lambda(z'_0) \frac{\partial \bar{T}(z'_0)}{\partial z}.$$

The results of calculations of $T(z)$ are given in fig.3.

Data for \bar{q} were taken from F.S. Johnson's study [4], the data on the dependence of function $A(z)$ on altitude were taken from M. Nicolet's work [5].

As there are no accurate data on the magnitude of heat fluxes from the upper boundary of the region under study ($h = 500$ km), outer fluxes of 0, 0.1, 0.5, and 1.0

erg/cm² sec have been considered.

As we can see from fig.3, consideration of the outer heat fluxes entering the earth's atmosphere greatly affects temperature distribution.

So, for instance, for $\lambda \sim T^{1/2}$, with the outer flux varying from

0.1 erg/cm² sec to 1 erg/cm² sec, the temperature at the 200-km level varies from 1150° to 1910°; at the 300-km level, it varies from 1400°K to 2780°K; and at the 500-km level, from 1500°K to 3790°K.

We know that the difficulty of determining $T(z)$ in the thermosphere is principally associated with the absence of reliable data on the composition of this region.

In this respect it is interesting to calculate temperature distributions for varying distributions of atmospheric components with altitude.

For the same composition variant, the change in the temperature curve, caused by the difference in the values of the outer fluxes as well as by the difference in the dependence of the coefficient of thermal con-

ductivity on temperature ($\lambda \sim T^{1/2}$; $\lambda \sim T^{3/4}$), preserves the same features as in the distribution shown in fig.3.

The difference in the temperature curve caused by the difference in the initial data on the composition of the upper atmosphere amounts to 10-15 percent.

2. Determination of Temperature from the Equation of Hydrostatic Equilibrium and From the Main Equation of an Ideal Gas

The second method of determining $T(z)$ is based on the equation of hydrostatic equilibrium

$$dp = -\rho dz \quad (23)$$

and on the equation of state of an ideal gas

$$p = \sum p_i = nkT. \quad (24)$$

$$(p = \sum_i m_i n_i = \bar{m} \cdot n; \quad n = \sum_i n_i).$$

Equations (23) and (24) give the relationship between density and temperature

$$\rho(z) = \rho(z_0) \frac{\bar{m}(z_0)}{\bar{m}(z)} \frac{T(z)}{T(z_0)} e^{-\int_{z_0}^z \frac{\bar{m}(z)g(z)}{kT} dz}, \quad (25)$$

This shows that empirical density data make it possible to obtain information on temperature distribution only if the distribution of the mean molecular mass $\bar{m}(z)$ (or the mean molecular weight $\bar{M} = N\bar{m}$) with altitude is known.

Equations (23) and (24) were used by O.P. Filipovich to solve the two following problems:

- a. Finding the molecular temperature from data on atmospheric density at altitudes above 100 km;
- b. Finding the true temperature from atmospheric density data (the last problem is concerned with correcting D.R. Bates' model [7] according to which isothermal conditions are found at altitudes exceeding approximately 200 km).

To find the molecular temperature, the method of the immediate integration of the equation of hydrostatic equilibrium

$$p(z) = p(z_0) - \int_{z_0}^z \rho(z)g(z)dz, \quad (26)$$

was used, which, together with (24), gives:

$$T_m(z) = \frac{\rho(z_0)}{\rho(z)} T_m(z_0) - \frac{\bar{m}(z_0)}{\rho(z)k} \int_{z_0}^z \rho(z)g(z)dz. \quad (27)$$

However, formula (27) cannot be used to determine $T_m(z)$ in practice.

This is due to the fact that the first, and most important, member in (27) is extremely sensitive to the minor mistakes in $\rho(z_0)$ or $T_m(z_0)$.

This difficulty can be eliminated if the equation of hydrostatic equilibrium is integrated from z_1 to z , where z_1 is the upper boundary, and not from z_0 to z .

Then instead of (27) we have

$$T_m(z) = \frac{\rho(z_1)}{\rho(z)} T_m(z_1) + \frac{\bar{m}(z)}{\rho(z)k} \int_z^{z_1} \rho(z) g(z) dz. \quad (28)$$

In expression (28) the second member plays the main role as the first member decreases exponentially with decreasing altitude.

The results of calculations of the molecular temperature from equation (28) are presented in fig.4.

As we can see from fig.4, the distribution of $T_m(z)$ differs considerably from the distribution of $T_m(z)$ obtained by the ARDC-1959 model. (Note that we proceeded from the distribution of $\rho(z)$ according to H. Kallman's model [8]).

The results of calculations of the true temperature $T(z)$ (the lower combination of curves) are also presented in fig.4.

To determine $T(z)$, we used the distribution of $T_m(z)$ thus obtained and the data on variations of the molecular weight with altitude given in L.E. Miller's study [9]. (The obtained distribution of $T(z)$ is, generally speaking, of no interest because, in our opinion, the data on L.E. Miller's $m(z)$ curve are not reliable.)

Let us now find the true temperature by other methods.

In this case the main purpose of calculating $T(z)$ was to determine whether D.R. Bates' [7] conclusion about the presence of isothermal conditions above 200 km is correct, without attempting to obtain more accurate results than those obtained by other authors.

This conclusion follows theoretically from the absence of a heat flux from the outer boundary of the upper atmosphere.

In accordance with our purpose, we proceeded from the assumption that there is a diffusion equilibrium among the main atmospheric components (O_2 , O , N_2) above the 120-km level, as assumed by D.R. Bates,

and from his boundary values for the temperature and concentration of the different components of the atmosphere.

In calculating $T(z)$, we proceeded from the values of density $\rho(z)$ given by H. Kallman's model [8].

We did not take, as D.R. Bates did, the definite analytical form for the temperature curve which would have certainly provided for isothermal conditions at high altitudes, but immediately determined the temperature agreeing best with the experimental values of density $\rho(z)$.

To do this, we divided the altitude interval under consideration (from 120 km $\leq z \leq$ 400 km) into layers 5-10 km thick and approximated its temperature curve by the linear function

$$T(z) = T(z_0) + \alpha (z - z_0), \quad z_0 \leq z \leq z_1, \quad (29)$$

where z_0 is the lower boundary of the layer, z is the upper boundary,

$\alpha = \frac{dT}{dz}$ is the temperature gradient, which is constant for the given layer.

Using, as D.R. Bates did, the equation of diffusion equilibrium for each component $(d_{pi} = \rho_i g dz)$, the equation for the state of an ideal gas $P_i = n_i kT$, and condition (29), we get

$$\rho(z) = \sum_i m_i n_i(z_0) \left(1 + \frac{\alpha(z - z_0)}{T(z_0)} \right)^{-\left(\frac{m_i g(z)}{k\alpha} + 1 \right)} \quad (30)$$

Equation (30) was solved by the sampling method which consists in selecting for each layer a value for gradient α at which density $\rho(z)$, calculated from formula (30), agrees best with the empirical value for $\rho(z)$.

The results of the calculations are given in fig.5.

In addition to the curve for $T(z)$, obtained from equation (30), fig.5 gives the curves obtained from the thermal conductivity equation (for a heat flux of 0 and 0.1 erg/cm² sec), as well as the temperature distribution curves of D.R. Bates and those based on the ARDC-1959 model and A.A. Pokhunov's [10] model.

As we can see from fig.6, the curve for $T(z)$ obtained by us differs from that of D.R. Bates [7] and corresponds to a slow temperature increase with altitude.

The following conclusions can be derived when comparing the curves obtained from the equation of hydrostatic equilibrium and the thermal conductivity equation.

The temperature calculated from the thermal conductivity equation increases with altitude more steadily and slowly up to an altitude of 300 km than the temperature calculated from the equation of hydrostatic equilibrium.

This difference may be due, on the one hand, to the fact that solar energy ($q, (z)$) is absorbed in a narrower altitude interval than that assumed by F.S. Johnson, and, on the other hand, to the fact that F.S. Johnson's values for $q(z)$ (which we used to calculate $T(z)$ from the thermal conductivity equation) are somewhat too low.

At altitudes above 300 km, the distribution of $T(z)$, obtained from the equation of hydrostatic equilibrium, agrees closely with that obtained from the thermal conductivity equation for an outer heat flux of $0.1 \text{ erg/cm}^2 \text{ sec}$.

For the convenience of comparison of our results with those of other authors, fig.6 gives the temperature distribution curves based on different models.

As we can see from fig.6, there are considerable discrepancies in results of temperature calculations based on the models of different authors.

These discrepancies can be attributed to differences in the methods used to construct the models, to the initial assumptions made concerning the diffusion equilibrium level, to the assumed power of particle dissociation at different levels, and also to the difference in the initial data on the distribution of atmospheric density with altitude.

It is only natural that models based on rocket and satellite data are of especial interest at the present time.

However, we must emphasize that until reliable data on the composition of the upper atmosphere are obtained, it will be difficult to say which model is preferable.

In fact, any model answering the empirical values for density $\rho(z)$ may be considered practical.

From this point of view the value of constructing models lies in that all possible variations of the composition of the upper atmosphere which give the same distribution for density $\rho(z)$ may be considered.

There is also a great difference between the temperature "sensitivity" toward changes in relative concentrations (i.e. in the composition of the atmosphere) in models constructed by means of the equation of hydrostatic equilibrium and the equation of state and in models constructed by means of the thermal conductivity equation.

This difference can apparently be explained by the difference in physical phenomena taken into consideration in the two equations used in the two methods.

Then, the curve corresponding to an outer heat flux of $0.1 \text{ erg/cm}^2 \text{ sec}$ is similar in nature to the curve for $T(z)$ in A.A. Pokhunov's model [10] constructed to an altitude of 200 km.

It should be pointed out that A.A. Pokhunov's distribution of $T(z)$ is based on direct measurements of the neutral atmospheric composition and does not make any assumptions as to the composition of the atmosphere.

In conclusion, let us note that the temperature in the ARDC-1959 model increases with altitude at the same gradient (0.5 degree/km) as in our curve obtained by means of the equation of hydrostatic equilibrium.

The higher absolute temperatures in the ARDC-1959 model, compared with ours, can be attributed to differences in basic assumptions about the composition of the atmosphere.

Literature

1. K. Y. Kondrat'yev: Radiative Heat Exchange in the Atmosphere. Gidrometeoizdat. 1956.
2. K. Y. Kondrat'yev: Thermal Regime of the Upper Atmosphere. O. P. Filipovich: Gidrometeoizdat, 1960.
3. K. Y. Kondrat'yev: Meteorological Investigations by Means of Rockets and Satellites. Gidrometeoizdat, 1962.
4. F. S. Johnson: Temperature in the High Atmosphere. Ann. de Géophys., v. 14, No. 1, 1958.
5. O. P. Filipovich: Calculations of the Temperature Distribution in the Upper Atmosphere. Vestnik L.G.U. (Leningrad University), Series of Physics and Chemistry, Vyp. 3, No. 16, 1961.
6. M. Nicolet: Thermosphere. Ann. de Géophys., 15, No. 1, 1959.
7. D. R. Bates: Some Problems Concerning the Terrestrial Atmosphere above about the 100 km Level. Proc. Roy. Soc., London, v. 253, No. 451, 1959.
8. H. Kallman: A Preliminary Model Based on Rocket and Satellite Data, Journ. Geophys. Res., v. 64, No. 4, No. 6, 1959.
9. L. E. Miller: Molecular Weight of Air at High Altitudes. Journ. Geophys. Res., v. 62, No. 2, 1957.
10. A. A. Pokhunov: Mass-spectrometrical Explorations of the Structural Parameters of the Earth's Atmosphere at Altitudes from 100 to 210 km. Artificial Earth Satellites, Vyp. 7, 1961.

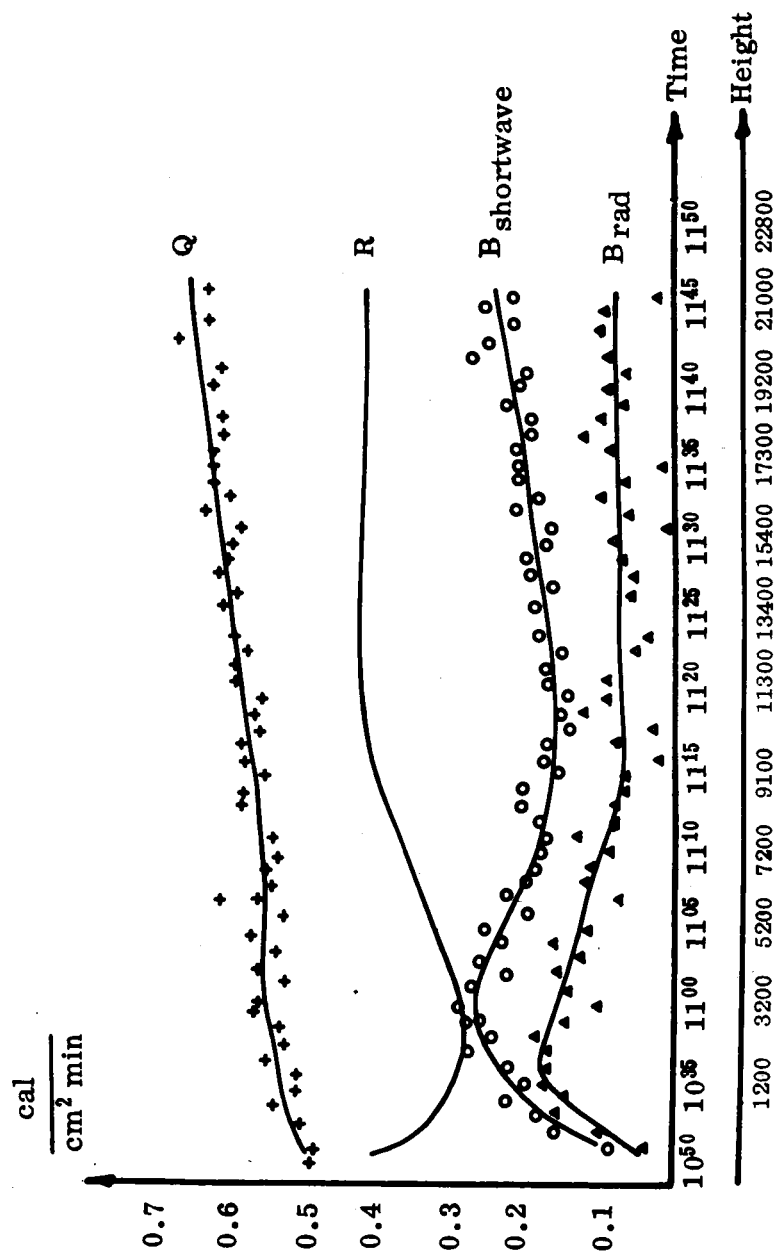


Fig. 1 — Vertical profile of the radiation balance and its components from observations on November 14, 1961.

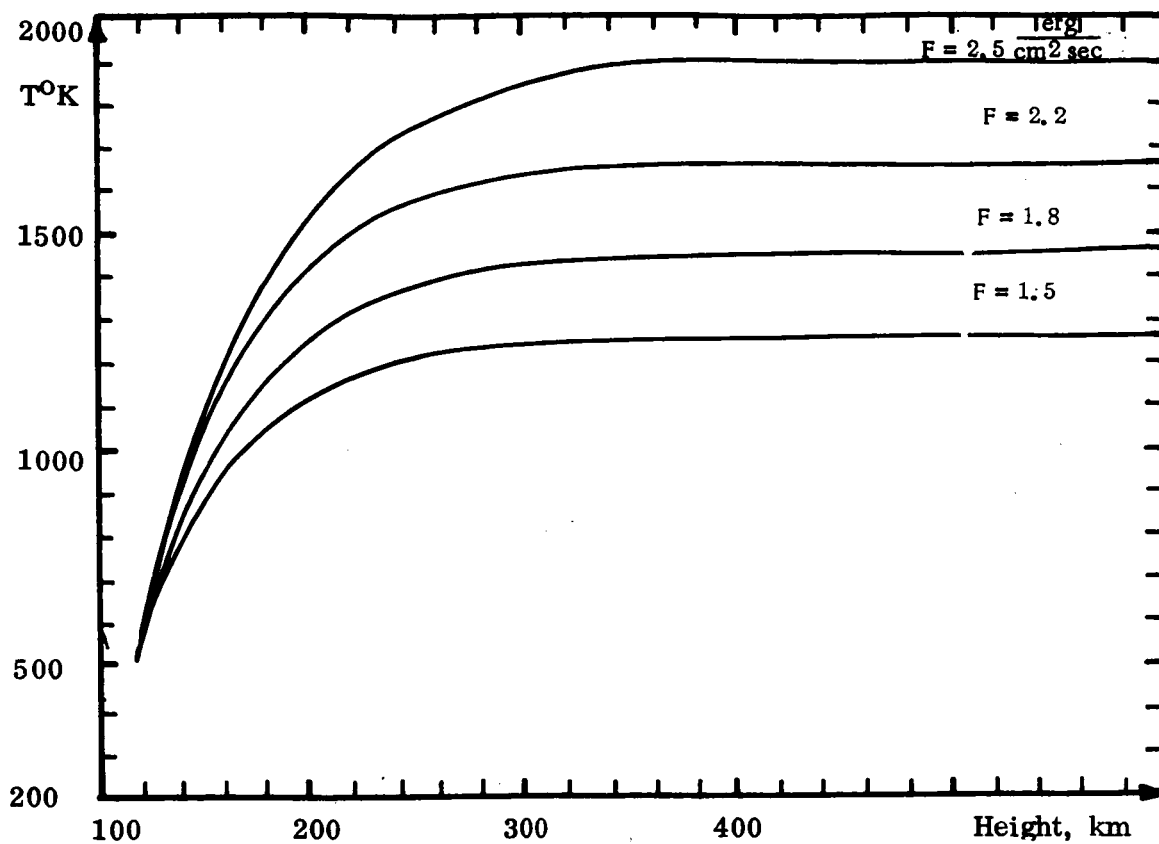


Fig. 2 — Results of calculations of vertical temperature distribution in the thermosphere with different values for radiant energy converted into thermal energy.

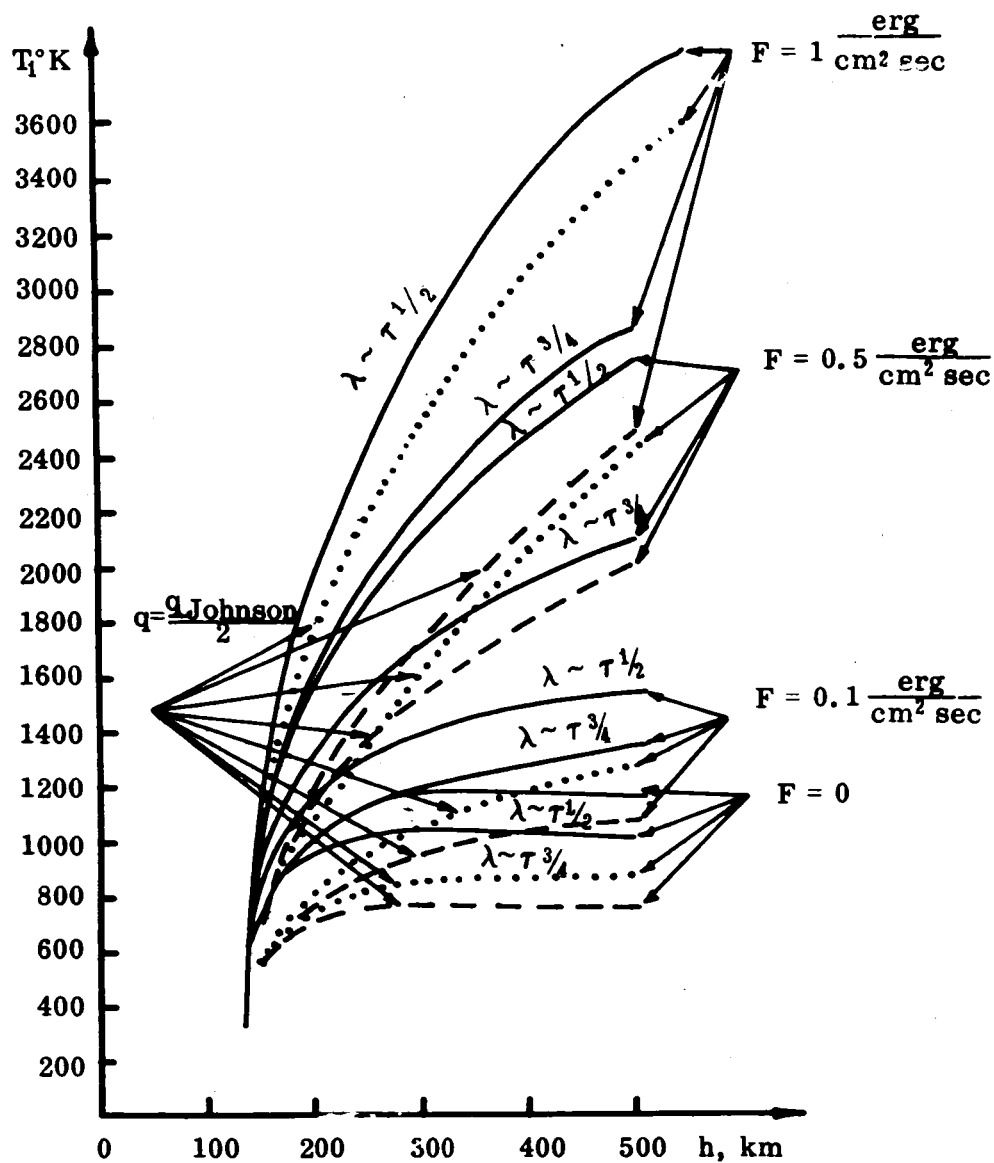


Fig. 3 — Results of calculations of temperature distribution with altitude. Four curves correspond to each of the four values of the energy flux $F(z_0)$:

Two curves for F.S. Johnson's values for \bar{q} : $\bar{q} = q_{\text{Johnson}}$ (from

which the continuous thin line corresponds to the dependence $\lambda \sim T^{3/4}$, and the continuous thick line to $\lambda \sim T^{1/2}$) and two curves for the value

$\bar{q} = \frac{q_{\text{Johnson}}}{2}$ (dotted line for the dependence $\lambda \sim T^{3/4}$ and the dashed line for $\lambda \sim T^{1/2}$).

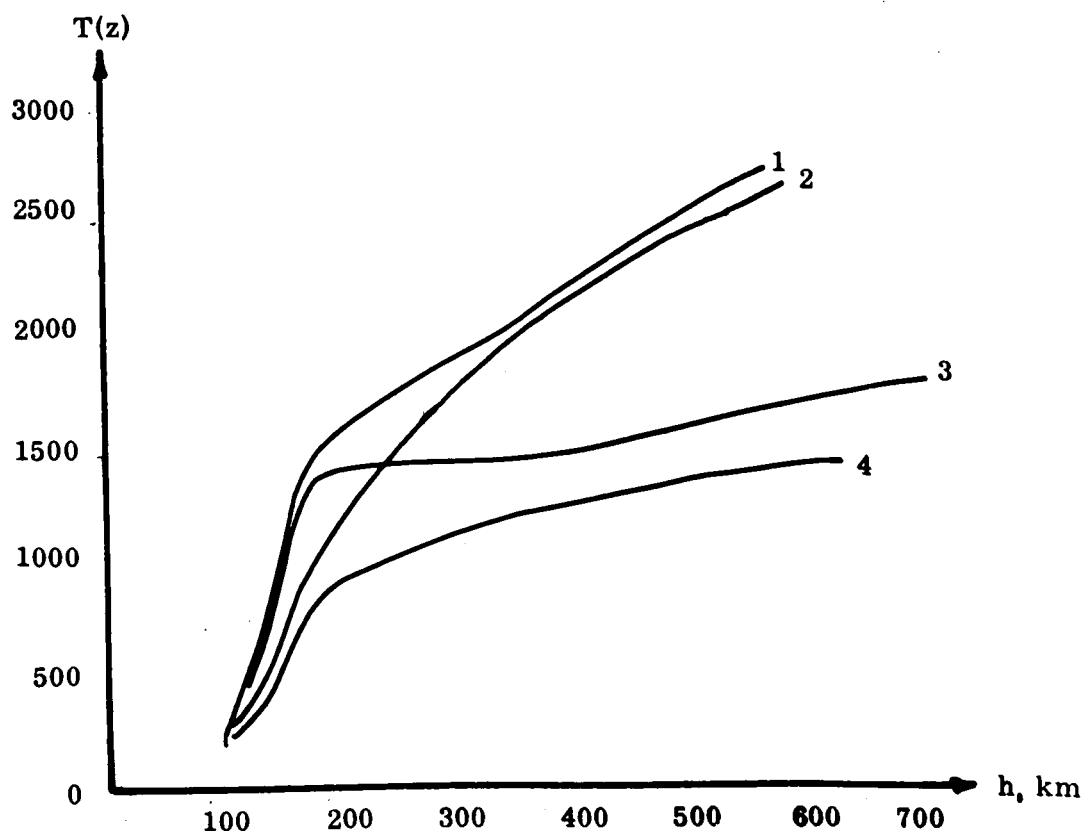


Fig. 4 — Comparison of the molecular temperature obtained from the equation of hydrostatic equilibrium with the ARDC-1959 model and of vertical distributions of true temperature.

1. Molecular temperature calculated according to (28).
2. Molecular temperature calculated according to ARDC-1959.
3. True temperature by ARDC-1959.
4. True temperature by (22) taking into consideration L. E. Miller's data on the chemical composition of the atmosphere.

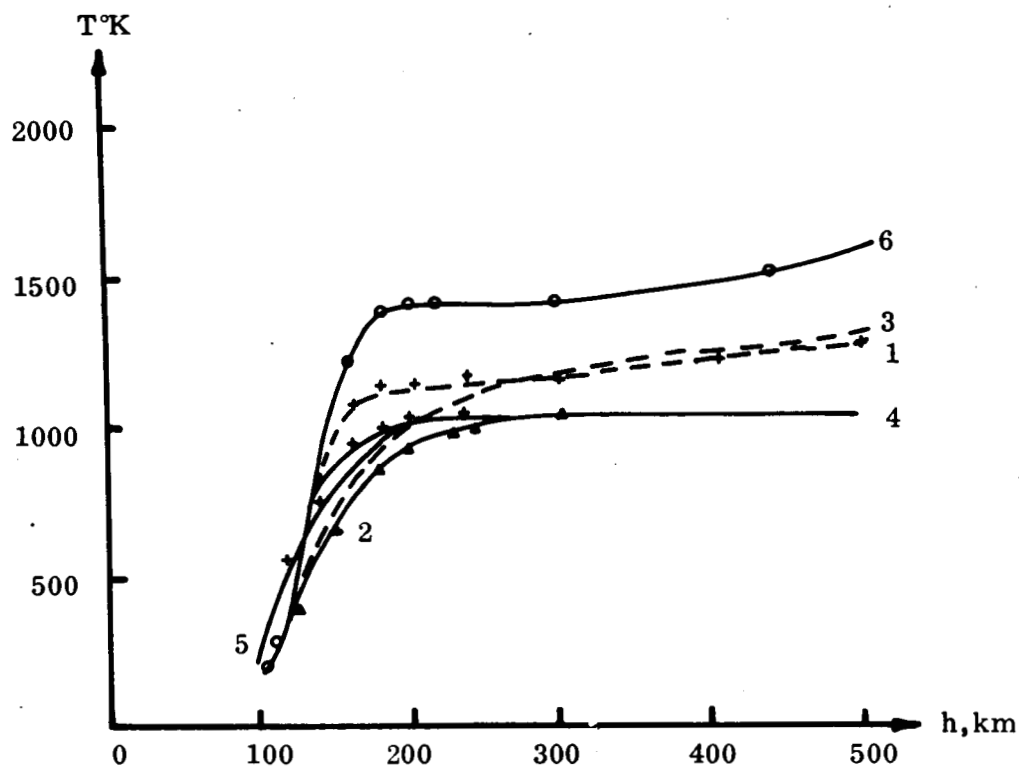


Fig. 5 — Comparison of the temperature distribution based on (30), according to the models of D.R. Bates, ARDC-1959, and

A.A. Pokhunov.

1 — according to (30); 2 — by D.R. Bates; 3 — from the thermal conductivity equation with $F = 0.1 \text{ erg/cm}^2 \text{ sec}$; 4 — the same with $F = 0$; 5 — by A.A. **Pokhunov**; 6 — by **ARDC-1959**.

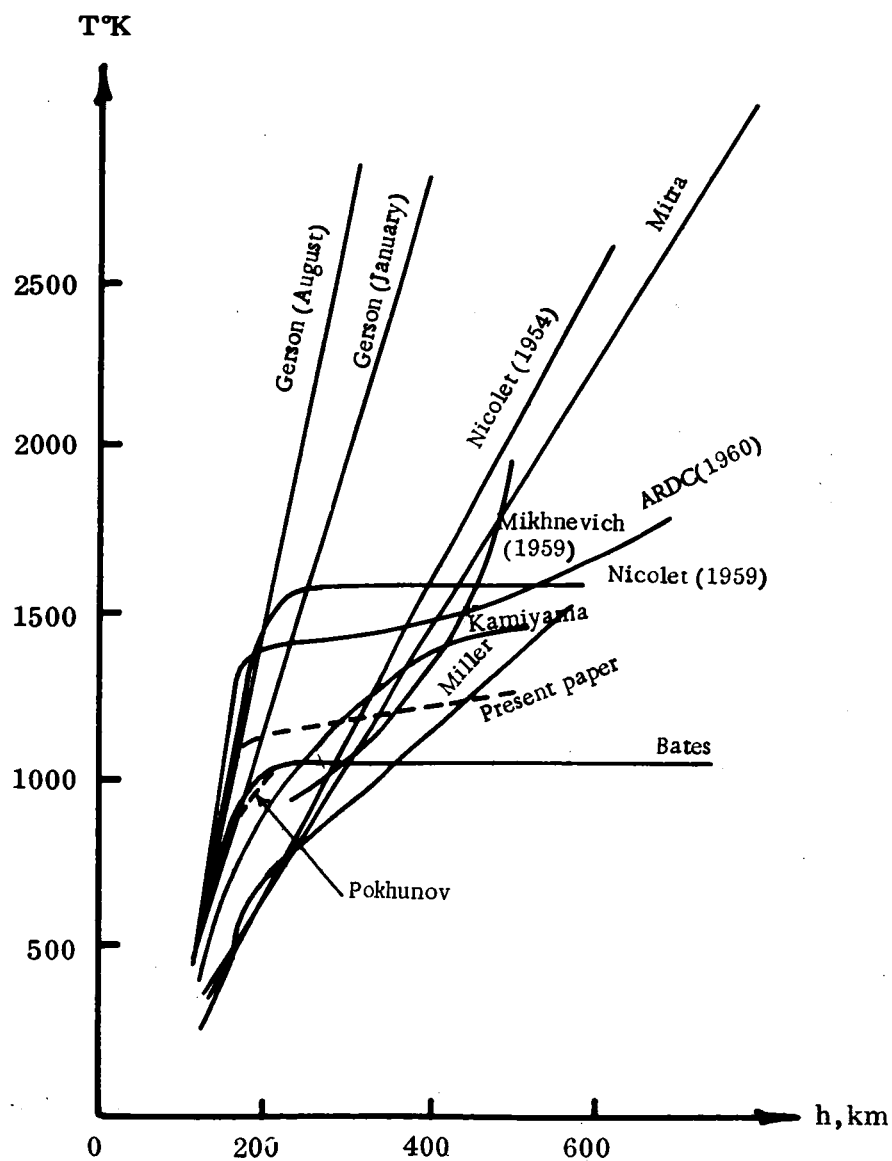


Fig. 6 — Temperature distributions corresponding to the different models of the atmosphere.